COLLOQUIUM

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Random and best approximation of convex bodies by polytopes

Abstract

Let K be a convex body in R^n and let $f : \partial K \to R_+$ be a continuous, positive function with $\int_{\partial K} f(x) d\mu_{\partial K}(x) = 1$ where $\mu_{\partial K}$ is the surface measure on ∂K. Let P_f be the probability measure on ∂K given by $dP_f(x) = f(x) d\mu_{\partial K}(x)$. Let κ be the (generalized) Gauß-Kronecker curvature and $E(f, N)$ the expected volume of the convex hull of N points chosen randomly on ∂K with respect to P_f. Then, under some regularity conditions on the boundary of K

$$\lim_{N\to\infty} \frac{\text{vol}_n(K) - E(f,N)}{\left(\frac{1}{N}\right)^{\frac{2}{n-1}}} = c_n \int_{\partial K} \frac{\kappa(x)^{\frac{1}{n-1}}}{f(x)^{\frac{2}{n-1}}} d\mu_{\partial K}(x),$$

where c_n is a constant depending on the dimension n only.

The minimum at the right-hand side is attained for the normalized affine surface area measure with density

$$f_{as}(x) = \frac{\kappa(x)^{\frac{1}{n+1}}}{\int_{\partial K} \kappa(x)^{\frac{1}{n+1}} d\mu_{\partial K}(x)}.$$

We will compare this random approximation to the best approximation.

372 Science and Engineering Building
Thursday, March 28th, 2002
3:00 to 4:00 P.M.
(Refreshment at 2:30 to 3:00 P.M. in Room 368,
Science and Engineering Building)

About the speaker